Problem 1: _________

Problem 2: _________

Problem 3: _________

Problem 4: _________

Total: _____________

Closed book, no calculators, and 1-side of an 8.5x11 inch paper of notes is allowed. Transform properties and pairs are provided. Cheating will not be tolerated.
Problem (1) DFTs

(a) The 2-Point DFT can be expressed by the following matrix equation.

$$\bar{X} = \bar{W} \bar{x}, \text{ where } \bar{W} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

What are the values (A, B, C, D) of the matrix?

If x[n] = [3, 1], what is X[k]?

A=1, B=1, C=1, D=-1

X[0] = 4, X[1] = 2

(b) The 6-point DFT of a real-valued signal, x[n], produces the sequence:

X[k] = [5, 3+jA, 5+j4, 2+jB, 5–j4, 3+j3]

What are the values for A and B?

A=-3, B=0
Problem (2) CTFT and DTFT Properties

(a) The real and positive-valued signal, \( x(t) \), has finite energy and is zero everywhere except for the range \( 0 < t < 4 \). The magnitude of the Fourier Transform of \( x(t) \) is given by:

\[
X(f) = 20e^{-f^2/10}
\]

What is the average value of \( x(t) \) in the range \( 0 < t < 4 \)?

\[\text{Average} = \frac{X(0)}{4} = 5\]

(b) The signal \( x_1[n] \) has the values:

\[
x_1[0] = 2, \quad x_1[1] = 4, \quad x_1[2] = -3, \quad x_1[3] = -10, \quad x_1[4] = 14, \quad x_1[5] = 2, \quad x_1[6] = -5, \quad x_1[7] = 4, \quad \ldots
\]

If \( x_1[n] \) \( \xrightarrow{\text{DTFT}} \) \( X_1(F) \); and \( X_2(F) = X_1(2F) \); what are the values of \( x_2[n] \) for \( n = 0 \) to \( 7 \)?

\[
x_2[n] = x_1[n/2]
\]

\[
x_2[0] = 2, \quad x_2[1] = 0, \quad x_2[2] = 4, \quad x_2[3] = 0, \quad x_2[4] = -3, \quad x_2[5] = 0, \quad x_2[6] = -10, \quad x_2[7] = 0
\]
Problem (3) Systems

(a) A continuous-time LTI system is described by the differential equation:

\[ 5y'''(t) - 6y''(t) + 2y(t) = 2x''(t) - 7x'(t) \]

Find the transfer function, \( H(s) \), and the frequency response, \( H(f) \).

\[
H(s) = \frac{2s^2 - 7s}{5s^3 - 6s^2 + 2}
\]

\[
H(f) = \frac{-8(\pi f)^2 - j14\pi f}{-j40(\pi f)^3 + 24(\pi f)^2 + 2}
\]

(b) A discrete-time LTI system is described by the following block diagram.

Find the transfer function, \( H(z) \), and the frequency response, \( H(F) \).

\[ y[n] = x[n] - 2y[n-1] + 4y[n-2] \]
\[ -4y[n-2] + 2y[n-1] + y[n] = x[n] \]

\[
H(z) = \frac{1}{-4z^2 + 2z^3 + 1}
\]

\[
H(F) = \frac{1}{-4e^{-j4\pi F} + 2e^{-j2\pi F} + 1}
\]
Problem (4) Pairs

Find the CTFT, \( X(f) \), of the following time-domain signals. Simplify any convolutions.

(a) \( x(t) = \text{rect}(t/5) \cdot \cos(2\pi 20t) \)
\[
X(f) = 5 \text{sinc}(5f) * \left\{ \frac{1}{2} \left( \delta(f + 20) + \delta(f - 20) \right) \right\} \\
X(f) = \frac{5}{2} \left\{ \text{sinc}(5(f + 20)) + \text{sinc}(5(f - 20)) \right\}
\]

(b) \( x(t) = \frac{d}{dt} \text{sinc}(2t - 4) \)
\[
X(f) = j 2\pi f \frac{1}{2} \text{rect} \left( \frac{1}{2} \right) e^{-j2\pi f^2} = j\pi f \text{rect} \left( \frac{1}{2} \right) e^{-j4\pi f}
\]

(c) \( x(t) = 4e^{j2\pi 5t} \cdot \sin(2\pi 3t) \)
\[
X(f) = 4 \left\{ \frac{1}{2} \left( \delta(f + 3 - 5) - \delta(f - 3 - 5) \right) \right\} \\
X(f) = 2j \left\{ \delta(f - 2) - \delta(f - 8) \right\}
\]